Spin $\frac{1}{2}$: korespondence $|\psi\rangle \in \mathcal{H}$ a $\vec{a} \in E^3$

nalezli jsme korespondenci

$$\vec{a} \longleftrightarrow |\psi\rangle = |\vec{a}:+\rangle \qquad \hat{\sigma}[\vec{a}]|\psi\rangle = |\vec{a}|\,|\psi\rangle \qquad |\vec{a}| = \langle\psi|\psi\rangle$$

$$\hat{\sigma}[\vec{a}] = 2|\psi\rangle\langle\psi| - \hat{\mathbb{1}}$$

normalizované stavy

$$\vec{e} \longleftrightarrow |\psi\rangle \qquad |\vec{e}| = \langle\psi|\psi\rangle = 1$$

$$\vec{e} = \cos\vartheta\, \vec{e}_z + \sin\vartheta\,(\cos\varphi\, \vec{e}_x + \sin\varphi\, \vec{e}_y)$$

$$|\psi\rangle = \cos\frac{\vartheta}{2}\, |\vec{e}_z +\rangle + \sin\frac{\vartheta}{2}\, e^{-i\varphi}\, |\vec{e}_z -\rangle$$

Blochova sféra : korespondence $\hat{D} \in \mathrm{Lin}\,\mathcal{H}$ a $\vec{a} \in E^3$

smíšený stav $\leftrightarrow$ operátor hustoty $\hat{D}$

$\hat{D}$ pozitivně definitní + normalizace: $\mathrm{Tr}\,\hat{D} = 1$

spektrální rozklad

$$\hat{D} = p_+ \hat{P}_+ + p_- \hat{P}_- \qquad \hat{\mathbb{1}} = \hat{P}_+ + \hat{P}_- \qquad 1 = p_+ + p_- \qquad p_+ \geq p_-$$

$\hat{P}_\pm$ projektory na 1D podprostory

$$\hat{P}_\pm = |\vec{e}\,\pm\rangle\langle\vec{e}\,\pm| \qquad a = p_+ - p_- \quad \Rightarrow \quad p_\pm = \frac{1 \pm a}{2} \qquad 0 \leq a \leq 1$$

$$\Downarrow$$

$$\hat{D} = \frac{1}{2}\left(\hat{\mathbb{1}} + \hat{\sigma}[\vec{a}]\right) \qquad \vec{a} = a\vec{e}$$

korespondence

$$\vec{a} \longleftrightarrow \hat{D} = \frac{1}{2}\left(\hat{\mathbb{1}} + \hat{\sigma}[\vec{a}]\right)$$

konvexita

$$\hat{D}\left[\sum_i p_i \vec{a}_i\right] = \sum_i p_i \hat{D}[\vec{a}_i]$$

pro $\sum_i p_i = 1$